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D1-D3 (or $\bar{\mathrm{D}}3$) systems with fluxes

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Abstract: In this article we study D1-D3 (or $\bar{\mathrm{D}}3$) brane systems with generic constant electric and magnetic fluxes in IIB string theory. We work out all possible supersymmetric configurations and find out via T-duality all of them and corresponding supersymmetry conditions could be related to the supersymmetric intersecting D1-D1 pairs. And we do D1-D3 (or $\bar{\mathrm{D}}3$) open string quantization for a class of configurations. We find that there are many near massless states in NS sector for near-BPS configurations. Furthermore we calculate open string pair creation rate in generic nonsupersymmetric configurations.

Keywords: D-branes, String Duality.

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Contents

1. Introduction

D-branes and anti-D-branes are nonperturbative objects in string theory, carrying RR charges. A single BPS D-brane preserve half supersymmetries in flat spacetime. But if we put two different D-branes together, or let a D-brane be in a compact configuration, supersymmtry could be broken [1, 2]. One typical example is a Dp-anti-Dp system which breaks all the supersymmetries. However, it turns out that when one turns on the background flux on the D-branes, the supersymmetries could be recovered. It is an interesting issue to look for these configurations. In [3], Mateos and Townsend found that if one turned on suitable gauge fluxes on tubular D2-brane, the system can be supersymmetric. This is so-called 'supertube'. The configuration could be taken as the blow-up of D-particles and has no net D2 brane charge. Effectively such system may be simplified to a D2-anti-D2 system with fluxes [4]. This discovery led to a lot of study of supersymmetric Dp-(anti)-Dp system with background fluxes [5 − 8]. In particular, it has been found even in Dp-anti-Dp system, the system could be supersymmetric if one turns on suitable fluxes.

Another class of non-BPS brane configuration is Dp-Dq with $p - q \neq 0 \pmod 4$. In particular, D0-D2 system is remarkable. The straightforward calculation of the D0-D2 open string spectrum shows that the ground state is tachyonic, which means that the system is unstable and non-BPS. However, it has been shown in [9] that D0-D2 system is actually dual to the (F1,D1) bound state. In fact, the underlying picture is that through tachyon condensation D0-D2 system settle down to a D0-D2 bound state, with D0 being dissolved into D2. Moreover, if one considers the gauge fluxes on D-brane, the story become more interesting. In D0-D2 system with magnetic field, in the zero-slop limit, there could exist an infinite tower of near massless states in the open string spectrum. On the other hand, the system could be studied in the framework of noncommutative gauge theory. In [10], the authors showed that in D0-D2 system with large magnetic field the D-particle could be taken as the soliton in (2+1)-dimensional noncommutative Yang-Mills theory. The large tower of near massless states corresponds to the fluctuations around the soliton solution. And due to the existence of the large magnetic field, the system is actually near-BPS, and the tachyon condensation could be very well studied, as shown in [10]. Another interesting aspect is that there exist BPS configuration in Dp-Dq ($p \neq q$) system if suitable constant background magnetic fluxes were turned on [11, 12].

It would be interesting to see if there exist BPS configuration in Dp-(anti)-Dq system with generic background fluxes. As the first step, in this paper, we will pay attention to the system with parallel D1-brane and D3 (or $\bar{\text{D}}3$)-brane in flat spacetime background. We will turn on constant fluxes on them, including generic electric and magnetic fluxes. We will try to find the most general supersymmetric configurations by using the Γ matrix method [13, 14]. We get the necessary conditions that the fluxes must satisfy. In two simplest setup, we obtain the sufficient condition directly. For more complicated cases, we try to attack the problem in another way. We find all possible supersymmetric configurations, which are related to the two simple cases via T-duality and Lorentz transformation. We also find that the supersymmetric configurations are equivalent to the systems studied in [5].

Besides looking for BPS configurations, there are other interesting issues to address in fluxed D1-D3 system. For generic flux setup, the system is nonsupersymmetric. The first step to investigate the nonsupersymmetric configurations is to do quantization of the open string between D1 and D3 (or $\bar{\text{D}}3$). This is a quite difficult problem due to perplexing boundary conditions imposed at the ends of the open string. The excitations on the string could have non-integer (or non-half integer) or even complex modes. There could be tachyonic excitation and it would be interesting to study the tachyon condensation in the system. In the D0-Dp system with constant magnetic fields, it has been found that there could exist large number of near-massless states if one tune the fluxes carefully so that the system is near-BPS [15, 11]. For the cases in the paper, we will show that this phenomenon also happen. Another interesting issue is the open string pair production [16, 17]. This will happen when the open string between D1 and D3 (or $\bar{\text{D}}3$) have complex modes. We will calculate the rate of string pair creation from one-loop vacuum amplitude of 1-3 strings.

This paper is organized as follows. In section 2, we will introduce the system and work out the supersymmetric configurations. First, we use Γ matrix to discuss supersymmetry conditions. We find necessary conditions for general systems and sufficient conditions for some simplified model. After that, via T-duality and Lorentz transformation, we will finish supersymmetry discussions and obtain all possible supersymmetric configurations in fluxed D1-D3 ($\bar{\text{D}}3$) system. Moreover, we will study the relation of fluxed D1-D3 system with D-string at angles with relative motion. In section 3, we will do mode expansion and quantization of open strings stretched between D1 and D3 (or $\bar{\text{D}}3$). In 3.1, we will calculate open string pair creation rate when there exist complex excitation modes. In section 3.2, We will determine GSO projection using in section 3.1. And we will study the open string spectrum when the system is near BPS. In section 4, we will give conclusions and discussions. In appendix A, we will present how to get all the necessary supersymmetry conditions in section 2. In appendix B and C, we will give details of T-dual discussions, mode expansions and quantization.

2. Supersymmetric configurations

We would like to study the D1-D3 ($\bar{\mathrm{D}}$3)-brane system in flat spacetime background. Let D1-brane lie along X^0, X^1 and D3 (or $\bar{\mathrm{D}}$3)-brane along $X^0, \dots X^3$. We will turn on all possible constant fluxes. On D1-brane, there is only electric flux:

$$\tilde{F}_{D1} = \frac{1}{2\pi\alpha'} \begin{pmatrix} 0 & -\tilde{E} \\ \tilde{E} & 0 \end{pmatrix}. \tag{2.1}$$

On D3 (or $\bar{\mathrm{D}}$3)-brane, we can turn on three electric fluxes and three magnetic fluxes. But using rotational symmetry, we can let electric fluxes on D3 (or $\bar{\mathrm{D}}$3) be only in planes X^0-X^1 and X^0-X^2 without losing generality:

$$F_{D3\ (\bar{D}3)} = \frac{1}{2\pi\alpha'} \begin{pmatrix} 0 & -E_1 & -E_2 & 0 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ 0 & B_2 & -B_1 & 0 \end{pmatrix} \tag{2.2}$$

The corresponding DBI action of D1 and D3 are respectively[1]

$$\mathscr{L}_1 = \sqrt{-\det(g + \tilde{F}_{D1})} = \sqrt{1 - \tilde{E}^2} \tag{2.3}$$

and

$$\begin{aligned} \mathscr{L}_2 &= \sqrt{-\det(g + F_{D3\ (or \bar{D}3)})} \\ &= \sqrt{1 - E_1^2 - E_2^2 + B_1^2 + B_2^2 + B_3^2 - (E_1B_1 + E_2B_2)^2} \end{aligned} \tag{2.4}$$

In this paper, we do not discuss critical cases when $\mathscr{L}_1 = 0$ or $\mathscr{L}_2 = 0$, so we require that

$$1 - \tilde{E}^2 > 0 \tag{2.5}$$

and

$$1 - E_1^2 - E_2^2 + B_1^2 + B_2^2 + B_3^2 - (E_1B_1 + E_2B_2)^2 > 0 \tag{2.6}$$

According to [13, 14], the conditions for supersymmetry is that there exist nonzero ϵ satisfying both

$$\begin{aligned} \Gamma^{(1)}\epsilon &= \epsilon, \\ \Gamma^{(2)}\epsilon &= \pm\epsilon, \end{aligned} \tag{2.7}$$

[1] In the following part of this article, except somewhere in section 3, we will let $2\pi\alpha' = 1$.

where $+$ is for D3, $-$ is for $\bar{\text{D}}3$, and $\Gamma^{(1)}$ and $\Gamma^{(2)}$ are the Gamma matrices for D1 and D3 respectively

$$\Gamma^{(1)} = \frac{1}{\sqrt{1-\tilde{E}^2}} \begin{pmatrix} 0 & \Gamma_{01}-\tilde{E} \\ \Gamma_{01}+\tilde{E} & 0 \end{pmatrix} \tag{2.8}$$

$$\begin{aligned} \Gamma^{(2)} &= \frac{1}{\sqrt{1-E_1^2-E_2^2+B_1^2+B_2^2+B_3^2-(E_1B_1+E_2B_2)^2}} \\ &\quad\times\left[\begin{pmatrix} 0 & K \\ K & 0 \end{pmatrix} + \begin{pmatrix} 0 & \Gamma_{0123}-E_1B_1-E_2B_2 \\ -\Gamma_{0123}+E_1B_1+E_2B_2 & 0 \end{pmatrix}\right] \end{aligned} \tag{2.9}$$

with

$$K = -E_1\Gamma_{23} + E_2\Gamma_{13} + B_1\Gamma_{01} + B_2\Gamma_{02} + B_3\Gamma_{03}. \tag{2.10}$$

Because IIB theory is chiral, ϵ must also satisfy

$$\tilde{\Gamma}_{11}\epsilon = \begin{pmatrix} \Gamma_{11} & 0 \\ 0 & \Gamma_{11} \end{pmatrix} \epsilon = \epsilon. \tag{2.11}$$

It would be convenient to let

$$\epsilon = \begin{pmatrix} \epsilon' \\ \epsilon'' \end{pmatrix}. \tag{2.12}$$

From(2.7), we can deduce that

$$[\Gamma^{(1)}, \Gamma^{(2)}]\epsilon = 0, \tag{2.13}$$

which leads to

$$\begin{aligned} (\tilde{E}E_1\Gamma_{23} + E_2\Gamma_{03} - \tilde{E}E_2\Gamma_{13} - \tilde{E}B_1\Gamma_{01} + B_2\Gamma_{12} - \tilde{E}B_2\Gamma_{02} \\ +B_3\Gamma_{13} - \tilde{E}B_3\Gamma_{03} - \Gamma_{23} + (E_1B_1+E_2B_2)\Gamma_{01})\epsilon' = 0, \end{aligned} \tag{2.14}$$

and

$$\begin{aligned} (-\tilde{E}E_1\Gamma_{23} + E_2\Gamma_{03} + \tilde{E}E_2\Gamma_{13} + \tilde{E}B_1\Gamma_{01} + B_2\Gamma_{12} + \tilde{E}B_2\Gamma_{02} \\ +B_3\Gamma_{13} + \tilde{E}B_3\Gamma_{03} + \Gamma_{23} - (E_1B_1+E_2B_2)\Gamma_{01})\epsilon'' = 0. \end{aligned} \tag{2.15}$$

In order to have nonzero solution to the equation (2.13), one of the equations (2.14), (2.15) must have nonzero solution.

Let

$$A\epsilon' = 0 \tag{2.16}$$

denote the equation (2.14). If the equation

$$A^2\epsilon' = 0 \tag{2.17}$$

do not have nonzero solutions, the equation (2.14) also do not have nonzero solutions. The equation (2.17) gives

$$\begin{aligned}0 = &[-(\tilde{E}E_1-1)^2+(\tilde{E}B_1-E_1B_1-E_2B_2)^2+(E_2-\tilde{E}B_3)^2-B_2^2\\&-(\tilde{E}E_2-B_3)^2+\tilde{E}^2B_2^2]\epsilon'\\&+2[(\tilde{E}E_1-1)(-\tilde{E}B_1+E_1B_1+E_2B_2)+B_2(E_2-\tilde{E}B_3)\\&\quad-\tilde{E}B_2(\tilde{E}E_2-B_3)]\Gamma_{0123}\epsilon'.\end{aligned} \tag{2.18}$$

Because $(\Gamma_{0123})^2=-\mathrm{I}$, so Γ_{0123} only have eigenvalues $\pm i$. Thus the necessary condition for equation (2.18) to have nonzero solutions is the constant term and the coefficient of Γ_{0123} on its right hand side must be zero simultaneously. This gives us two equations

$$\begin{aligned}0 &= -(\tilde{E}E_1-1)^2+(1-\tilde{E}^2)(E_2^2-B_2^2-B_3^2)+(\tilde{E}B_1-E_1B_1-E_2B_2)^2,\\0 &= (\tilde{E}E_1-1)(\tilde{E}B_1-E_1B_1-E_2B_2)-(1-\tilde{E}^2)E_2B_2,\end{aligned} \tag{2.19}$$

which should hold simultaneously. The similar analysis on the equation (2.15) leads to the same conditions (2.19). If the conditions (2.19) cannot be satisfied by the fluxes, the equations (2.14), (2.15) have no nonzero solution, so the configurations can not be supersymmetric. In other words, the equations in (2.19) are the necessary condition for supersymmetry. Moreover, the fluxes must respect the inequities (2.5), (2.6).

In appendix A, we prove that (2.19) only have two solutions which do not break (2.5) and (2.6).

- One solution is

$$\begin{aligned}&-1<\tilde{E}=E_1<1, \qquad B_1\neq\frac{E_1E_2B_2}{1-E_1^2}, \qquad 1<E_1^2+E_2^2<1+B_3^2,\\&B_2=\pm\sqrt{\frac{(1-E_1^2)(1-E_1^2-E_2^2+B_3^2)}{E_1^2+E_2^2-1}}.\end{aligned} \tag{2.20}$$

- The other solution is

$$\begin{aligned}&-1<\tilde{E}=E_1<1, \qquad B_1\neq\frac{E_1B_2}{E_2}, \qquad B_3=0,\\&E_2=\pm\sqrt{1-E_1^2}.\end{aligned} \tag{2.21}$$

These two solutions are just the necessary conditions for supersymmetric configurations. We will show that they are also sufficient conditions if we choose right sign for $B_1-\frac{E_1E_2B_2}{1-E_1^2}$ or $B_1-\frac{E_1B_2}{E_2}$. As the first step to prove the sufficiency, we will directly use Γ matrix method to study two simple cases. We will show that the general solutions (2.20), (2.21) could be related to these two simple cases via T-duality and Lorentz transformation. In these two simple cases, we will let $\tilde{E}=E_1=0$, and let $B_2=0$ in the second case (2.21). Now the equation (2.13) is

$$\begin{pmatrix} m_{11} & 0\\ 0 & m_{22}\end{pmatrix}\epsilon=0. \tag{2.22}$$

where

$$
\begin{aligned}
m_{11} &\equiv E_2\Gamma_{03} + B_2\Gamma_{12} + B_3\Gamma_{13} - \Gamma_{23} + E_2B_2\Gamma_{01}, \\
m_{22} &\equiv E_2\Gamma_{03} + B_2\Gamma_{12} + B_3\Gamma_{13} + \Gamma_{23} - E_2B_2\Gamma_{01}.
\end{aligned} \tag{2.23}
$$

(i) Case 1: $\tilde{E} = E_1 = 0$, $B_3 \neq 0$, other fluxes satisfy (2.20)
In this case, one can multiply

$$
\begin{pmatrix} \Gamma_{13} & 0 \\ 0 & \Gamma_{13} \end{pmatrix}
$$

on the equation (2.22) and obtain

$$
M\epsilon = \epsilon, \tag{2.24}
$$

where M is

$$
\frac{1}{B_3}\begin{pmatrix} E_2\Gamma_{01} + B_2\Gamma_{23} + \Gamma_{12} - E_2B_2\Gamma_{03} & 0 \\ 0 & E_2\Gamma_{01} + B_2\Gamma_{23} - \Gamma_{12} + E_2B_2\Gamma_{03} \end{pmatrix}
$$

Since the equations

$$
\Gamma^{(1)}\epsilon = \epsilon, \qquad M\epsilon = \epsilon \tag{2.25}
$$

imply that

$$
\Gamma^{(2)}\epsilon = \begin{cases} \epsilon, & \text{if } B_1 > 0 \\ -\epsilon, & \text{if } B_1 < 0 \end{cases} \tag{2.26}
$$

the condition (2.7) now is equivalent to (2.25) with $B_1 > 0$ for D1-D3 or $B_1 < 0$ for D1-$\bar{\text{D}}$3 system. It is easy to check that

$$
M^2 = \mathrm{I}, \qquad \mathrm{Tr}M = 0, \tag{2.27}
$$

$$
[M, \Gamma^{(1)}] = 0, \qquad \mathrm{Tr}(M\Gamma^{(1)}) = 0, \tag{2.28}
$$

and

$$
[M, \tilde{\Gamma}_{11}] = 0, \;\; \mathrm{Tr}(M\tilde{\Gamma}_{11}) = 0. \tag{2.29}
$$

From these properties, we can conclude that when the fluxes satisfy (2.20) with $\tilde{E} = E_1 = 0, B_1 > 0$, D1-D3 system preserve 1/4 supersymmetries, and when the fluxes satisfy (2.20) and $\tilde{E} = E_1 = 0, B_1 < 0$, D1-$\bar{\text{D}}$3 system preserve 1/4 supersymmetries.

(ii) Case 2: $\tilde{E} = E_1 = 0$, $B_2 = 0$, other fluxes satisfy (2.21)
In this case, $B_2 = B_3 = 0$, so the relation (2.13) is simplified to

$$
\begin{pmatrix} E_2\Gamma_{03} - \Gamma_{23} & 0 \\ 0 & E_2\Gamma_{03} + \Gamma_{23} \end{pmatrix}\epsilon = 0. \tag{2.30}
$$

Let

$$\begin{pmatrix} \Gamma_{23} & 0 \\ 0 & -\Gamma_{23} \end{pmatrix}$$

multiply the equation (2.30), we obtain

$$N\epsilon = \epsilon. \tag{2.31}$$

where

$$N = -E_2 \begin{pmatrix} \Gamma_{02} & 0 \\ 0 & -\Gamma_{02} \end{pmatrix}$$

Similarly, we also find that $\Gamma^{(1)}\epsilon = \epsilon$ and $N\epsilon = \epsilon$ imply

$$\Gamma^{(2)}\epsilon = \begin{cases} \epsilon, & \text{if } B_1 > 0 \\ -\epsilon, & \text{if } B_1 < 0 \end{cases} \tag{2.32}$$

Similar to the matrix M above, N satisfy

$$N^2 = \mathrm{I}, \qquad \mathrm{Tr}N = 0, \tag{2.33}$$

$$[N, \Gamma^{(1)}] = 0, \qquad \mathrm{Tr}(N\Gamma^{(1)}) = 0, \tag{2.34}$$

and

$$[N, \tilde{\Gamma}_{11}] = 0, \ \mathrm{Tr}(N\tilde{\Gamma}_{11}) = 0. \tag{2.35}$$

Therefore, when the fluxes satisfy (2.21) and $\tilde{E} = E_1 = 0, \ B_2 = 0, \ B_1 > 0$, D1-D3 system preserve 1/4 supersymmetries, and when the fluxes satisfy (2.21) and $\tilde{E} = E_1 = 0, \ B_2 = 0, \ B_1 < 0$, D1-$\bar{\text{D}}$3 system preserve 1/4 supersymmetries.

With the above detailed analysis of two simple supersymmetric configurations, let us turn to the general solutions (2.20) and (2.21). The key point is that since $E_1 = \tilde{E}$ both solutions could be related to the above two simple cases via T-duality and Lorentz transformation. We leave the details of the transformation to appendix B and just give the final result here. The D1-D3(or $\bar{\text{D}}$3) system with the fluxes satisfying (2.20) is actually equivalent to D1-D3(or $\bar{\text{D}}$3) system without electric field on D1 worldvolume and

$$F_{D3} = \frac{1}{2\pi\alpha'} \begin{pmatrix} 0 & 0 & -\hat{E}_2 & 0 \\ 0 & 0 & \hat{B}_3 & -\hat{B}_2 \\ \hat{E}_2 & -\hat{B}_3 & 0 & \hat{B}_1 \\ 0 & \hat{B}_2 & -\hat{B}_1 & 0 \end{pmatrix} \tag{2.36}$$

on D3 worldvolume, with $\hat{E}_2, \hat{B}_1, \hat{B}_2$ and $\hat{B}_3$ being given in (B.9). This is the same configuration we have discussed before. Besides the solution (2.20), the extra requirement for supersymmetry is $B_1 - \frac{E_1E_2B_2}{1-E_1^2} > 0$ for D1-D3 or $B_1 - \frac{E_1E_2B_2}{1-E_1^2} < 0$ for D1-$\bar{\text{D}}$3. Furthermore, since the electric field along X^1 direction vanishes, one can do one more T-duality along

X^1 to get a D0-D2 system with fluxes. And another T-duality leads to the equivalent supersymmetric intersecting D1-D1 configurations with relative angle and motion.

For the solutions (2.21), the similar treatment shows that the solutions are also sufficient condition for supersymmetry provided that $B_1 - \frac{E_1 B_2}{E_2} > 0$ for D1-D3 or $B_1 - \frac{E_1 B_2}{E_2} < 0$ for D1-$\bar{\text{D}}$3. Similarly the configurations could be related to fluxed D0-D2 system and intersecting D1-D1 at angle with relative motion. All the details on T-duality and equivalence with other configurations could be found in appendix B.

In summary, we have proved that

- when the fluxes satisfy (2.20) and $B_1 - \frac{E_1 E_2 B_2}{1-E_1^2} > 0$, or when the fluxes satisfy (2.21) and $B_1 - \frac{E_1 B_2}{E_2} > 0$, D1-D3 systems are supersymmetric.
- when the fluxes satisfy (2.20) and $B_1 - \frac{E_1 E_2 B_2}{1-E_1^2} < 0$, or when the fluxes satisfy (2.21) and $B_1 - \frac{E_1 B_2}{E_2} < 0$, D1-$\bar{\text{D}}$3 systems are supersymmetric.
- The supersymmetric D1-D3 configurations we have found keep one-quarter supersymmetries and are dual to the supersymmetric D1-D1 systems studied in [5].

3. Open string quantization and pair creation

In this section, we will study generic nonsupersymmetric configurations. We will discuss the open string excitations between D1 and D3(or $\bar{\text{D}}$3)-branes by doing quantization of the open string with boundary conditions, which are determined by the fluxes on the D-branes. The excitation modes could be real but not integer or half-integer, and even could also be complex. There are various interesting issues to address. We will mainly focus on the open string pair production and mass spectrum of near-BPS configurations.

As usual, the boundary conditions at the ends of the open string decide the modes expansion. In our case, the boundary conditions at two ends are different. At $\sigma = 0$ endpoint, we have boundary condition:

$$\begin{aligned}
0 &= \partial_\sigma X^0 + \tilde{E}\partial_\tau X^1, \\
0 &= \partial_\sigma X^1 + \tilde{E}\partial_\tau X^0, \\
0 &= \partial_\tau X^2, \\
0 &= \partial_\tau X^3.
\end{aligned} \tag{3.1}$$

While at $\sigma = \pi$ endpoint, we have

$$\begin{aligned}
0 &= \partial_\sigma X^0 + E_1\partial_\tau X^1 + E_2\partial_\tau X^2, \\
0 &= \partial_\sigma X^1 + E_1\partial_\tau X^0 + B_3\partial_\tau X^2 - B_2\partial_\tau X^3, \\
0 &= \partial_\sigma X^2 + E_2\partial_\tau X^0 - B_3\partial_\tau X^1 + B_1\partial_\tau X^3, \\
0 &= \partial_\sigma X^3 + B_2\partial_\tau X^1 - B_1\partial_\tau X^2.
\end{aligned} \tag{3.2}$$

Without losing generality, we let $X^2|_{\sigma=0} = X^3|_{\sigma=0} = 0$. In $4, 5, \ldots, 9$ directions, the usual Dirichlet boundary conditions are imposed. We let the distance of two branes be y in x^4 direction.

Now we do mode expansions for X^0, X^1, X^2, X^3 with ansatz

$$\begin{aligned}
X^\mu &= x_0^\mu + B_0^\mu \sigma - C_0^\mu \tau \\
&\quad + \sum_{r=n+A} \frac{ia_r^\mu}{r}(\mathrm{e}^{-ir(\tau-\sigma)} + \mathrm{e}^{-ir(\tau+\sigma)}) \\
&\quad + \sum_{r=n+A} \frac{ib_r^\mu}{r}(\mathrm{e}^{-ir(\tau-\sigma)} - \mathrm{e}^{-ir(\tau+\sigma)}) \\
&\quad + \cdots ,
\end{aligned} \tag{3.3}$$

where "$\cdots$" denote all possible other modes.

Imposing the boundary conditions (3.1), (3.2) to (3.3), we have

$$\begin{aligned}
b_r^0 &= \tilde{E} a_r^1, \\
b_r^1 &= \tilde{E} a_r^0, \\
a_r^2 &= a_r^3 = 0
\end{aligned} \tag{3.4}$$

and

$$\begin{aligned}
0 &= (1 - \tilde{E}E_1)(1 - \mathrm{e}^{-i2\pi A})a_r^0 + (\tilde{E} - E_1)(1 + \mathrm{e}^{-i2\pi A})a_r^1 \\
&\quad - E_2(1 - \mathrm{e}^{-i2\pi A})b_r^2, \\
0 &= (\tilde{E} - E_1)(1 + \mathrm{e}^{-i2\pi A})a_r^0 + (1 - \tilde{E}E_1)(1 - \mathrm{e}^{-i2\pi A})a_r^1 \\
&\quad - B_3(1 - \mathrm{e}^{-i2\pi A})b_r^2 + B_2(1 - \mathrm{e}^{-i2\pi A})b_r^3, \\
0 &= [-E_2(1 + \mathrm{e}^{-i2\pi A}) + \tilde{E}B_3(1 - \mathrm{e}^{-i2\pi A})]a_r^0 + [B_3(1 + \mathrm{e}^{-i2\pi A}) \\
&\quad - \tilde{E}E_2(1 - \mathrm{e}^{-i2\pi A})]a_r^1 + (1 + \mathrm{e}^{-i2\pi A})b_r^2 - B_1(1 - \mathrm{e}^{-i2\pi A})b_r^3, \\
0 &= -\tilde{E}B_2(1 - \mathrm{e}^{-i2\pi A})a_r^0 - B_2(1 + \mathrm{e}^{-i2\pi A})a_r^1 + B_1(1 - \mathrm{e}^{-i2\pi A})b_r^2 \\
&\quad + (1 + \mathrm{e}^{-i2\pi A})b_r^3.
\end{aligned} \tag{3.5}$$

If some fields really have r modes , the coefficient matrix of (3.5) must have zero determinant. This help us to fix A from the equation

$$\begin{aligned}
&(B_1 - \tilde{E}E_1B_1 - \tilde{E}E_2B_2)^2 \tan^4 \pi A \\
&+[-1 + E_2^2 - B_2^2 - B_3^2 + (E_1B_1 + E_2B_2)^2 \\
&\quad + \tilde{E}^2(-E_1^2 - E_2^2 + B_1^2 + B_2^2 + B_3^2) + 2\tilde{E}E_1 \\
&\quad - 2\tilde{E}B_1(E_1B_1 + E_2B_2)] \tan^2 \pi A \\
&\qquad\qquad - (\tilde{E} - E_1)^2 = 0
\end{aligned} \tag{3.6}$$

It is easy to check that when the fluxes satisfy (2.20) or (2.21), there exist only integer modes.

For simplicity, we let $\tilde{E} = E_1$ and other fluxes be free. This include the supersymmetric case, and also include many other nonsupersymmetric ones. Now the possible values for A are

$$A = 0 \tag{3.7}$$

which gives the integer modes, and also

$$(\tan\pi A)^2 = \frac{\Lambda}{\Delta^2}, \tag{3.8}$$

where

$$\begin{aligned}
\Lambda &= (1 - E_1^2 - E_2^2)(1 - E_1^2 + B_2^2) + (1 - E_1^2)B_3^2 && (3.9)\\
\Delta &= (1 - E_1^2)B_1 - E_1 E_2 B_2. && (3.10)
\end{aligned}$$

If $\Lambda > 0$, there will be real fractional excitation modes. We will discuss this case in subsection 3.2. If $\Lambda < 0$, then A is pure imaginary. Let us discuss this case first. In this case, it is convenient to introduce a real parameter ϵ,

$$\epsilon \equiv \frac{1}{\pi}\mathrm{arctanh}\frac{\sqrt{-\Lambda}}{\Delta}. \tag{3.11}$$

The sign of ϵ is the same as the sign of $\Delta \neq 0$.

The mode expansions of X^μ and its super-partner is quite involved. From them one can define the symplectic form to do quantization. After proper linear transformation, one can write the Hamiltonian in a canonical way. The details on the mode expansion and quantization could be found in appendix C.

The Hamiltonian of 1-3 string in $0, 1, 2, 3$ directions is

$$\begin{aligned}
H_{(0,1,2,3)} = \tfrac{1}{2}\textstyle\int_0^\pi \mathrm{d}\sigma\ (&\partial_\tau X^\mu \partial_\tau X_\mu + \partial_\sigma X^\mu \partial_\sigma X_\mu \\
&+ i\psi_+^\mu \partial_\sigma \psi_{\mu+} - i\psi_-^\mu \partial_\sigma \psi_{\mu-}). && (3.12)
\end{aligned}$$

Here $\mu = 0, 1, 2, 3$.

Due to the existence of non-diagonal terms, the Hamiltonian looks messy in terms of the original independent modes. In terms of the transformed modes, we obtain

$$\begin{aligned}
H_{0,1,2,3} = H_{0-mode} &+ \frac{1}{2}\sum_{n\neq 0} c_n c_{-n} + \frac{1}{2}\sum_{n\neq 0} d_n d_{-n} \\
&- \frac{1}{2}\sum_n b_{n+i\epsilon} b_{-n-i\epsilon} - \frac{1}{2}\sum_n b_{n-i\epsilon} b_{-n+i\epsilon} \\
&- \frac{1}{2}\sum_r r\phi_r \phi_{-r} - \frac{1}{2}\sum_r r\xi_r \xi_{-r} \\
&+ \frac{1}{2}\sum_r (r + i\epsilon)\beta_{r+i\epsilon}\beta_{-r-i\epsilon} + \frac{1}{2}\sum_r (r - i\epsilon)\beta_{r-i\epsilon}\beta_{-r+i\epsilon}. && (3.13)
\end{aligned}$$

In (3.13), H_{0-mode} comes from the zero-mode

$$\begin{aligned}
H_{0-mode} \equiv &-\frac{\pi}{2}(1 - E_1^2 - E_2^2)(C_0^0)^2 + \frac{\pi}{2}(1 - E_1^2 + B_2^2 + B_3^2)(C_0^1)^2 \\
&- \pi E_2 B_3 C_0^0 C_0^1, && (3.14)
\end{aligned}$$

and the (anti-)commutation relations between modes take the canonical form:

$$
\begin{aligned}
[c_n, c_m] &= n\delta_{n,-m}, \qquad [d_n, d_m] = n\delta_{n,-m}, \qquad [c_n, d_m] = 0,\\
[b_{n+i\epsilon}, b_{m-i\epsilon}] &= -(n+i\epsilon)\delta_{n,-m}.\\
\{\phi_r, \phi_s\} &= \delta_{r,-s}, \qquad \{\xi_r, \xi_s\} = \delta_{r,-s}, \qquad \{\phi_r, \xi_s\} = 0,\\
\{\beta_{r+i\epsilon}, \beta_{s-i\epsilon}\} &= -\delta_{r,-s}.
\end{aligned} \tag{3.15}
$$

Written in normal order, the Hamiltonian is

$$
\begin{aligned}
H_{0,1,2,3} = & \; H_{0-mode} + \sum_{n>0} c_{-n}c_n + \sum_{n>0} d_{-n}d_n - \sum_{n\geqslant 0} b_{-n-i|\epsilon|}b_{n+i|\epsilon|}\\
& - \sum_{n>0} b_{-n+i|\epsilon|}b_{n-i|\epsilon|} + \sum_{r>0} r\phi_{-r}\phi_r + \sum_{r>0} r\xi_{-r}\xi_r\\
& - \sum_{r\geqslant 0}(r+i|\epsilon|)\beta_{-r-i|\epsilon|}\beta_{r+i|\epsilon|} - \sum_{r>0}(r-i|\epsilon|)\beta_{-r+i|\epsilon|}\beta_{r-i|\epsilon|}\\
& + \mathcal{E}_0.
\end{aligned} \tag{3.16}
$$

where $\mathcal{E}_0$ is the zero-point energy

$$
\mathcal{E}_0 = \begin{cases} 0, & \text{R sector}\,, \\ \frac{i|\epsilon|}{2} - \frac{1}{4}, & \text{NS sector}\,. \end{cases} \tag{3.17}
$$

Taking into account of the excitations along other directions and ghosts, the vacuum state in NS sector $|0\rangle_{\rm NS}$ has energy

$$
\mathcal{E}_v = \frac{y^2}{2\pi} + \frac{i|\epsilon|}{2} - \frac{1}{2} = \frac{y^2}{2\pi} + \begin{cases} \frac{i\epsilon}{2} - \frac{1}{2} & \Delta > 0\,, \\ -\frac{i\epsilon}{2} - \frac{1}{2} & \Delta < 0\,. \end{cases} \tag{3.18}
$$

where Δ was defined as (3.10).

The GSO projection is quite subtle in the cases with background fluxes. There is spectral flow when the fluxes are varied [11, 12]. In our case, when $\Delta < 0$, the GSO projection on $|0\rangle_{\rm NS}$ is different from the case when $\Delta > 0$. From (C.2), (C.5), we find that $\beta^{\nu\pm}_{r\pm i|\epsilon|}$ ($\nu = 0, 1, 3$) have different relations with $\beta_{r\pm i|\epsilon|}$ when the signs of Δ are different. Thus when Δ change its sign, the normal ordering in (3.16) indicates that the orientation of D-brane has changed. As discussed in [11], the eigenvalues of GSO projection operator on $|0\rangle_{\rm NS}$ could be defined by the function $\frac{1+f(\Delta)}{2}$, where $f(\Delta)$ can only be ± 1, and must take opposite values when Δ changes sign. In D1-D3 systems, we will prove that

$$
f(\Delta) = \begin{cases} -1 & \Delta > 0\,, \\ 1 & \Delta < 0 \end{cases} \tag{3.19}
$$

in subsection 3.2. In D1-$\bar{\rm D}$3 systems, $f(\Delta)$ take opposite values to (3.19).

The Hamiltonian in (3.16) is not Hermitian due to the existence of the complex ground state energy. This is reminiscent of the open string in an electric field [16], and suggest quantum instability due to the open string pair production. On the other hand, despite the imaginary part of the ground state energy, the real part of the ground state energy could be negative, indicating the existence of tachyon and classical instability. As usual, the classical instability is more fatal to the system. With the GSO projection, we know that the ground state in D1-D3 system with $\Delta > 0$ will be projected out and the system seems to be classically stable. On the other hand, for the D1-$\bar{\text{D}}$3 system with $\Delta > 0$, the system is tachyonic. The D1 will dissolve into $\bar{\text{D}}$3 via tachyon condensation, while there may still be open string pair production after tachyon condensation. We will not study the tachyon condensation in this paper. We will focus on the open string pair production in the next subsection.

3.1 Open string pair creation

Now we can calculate 1-loop vacuum amplitude $\mathcal{A}$ for open strings between D1 and D3 (or $\bar{\text{D}}$3) branes:

$$\begin{aligned}\mathcal{A} = i\frac{\varphi_{\text{flux}}}{\alpha'^2}V_2\int_0^\infty dt\frac{q^{\frac{y^2}{4\pi^2\alpha'}}}{t^2\eta^9(it)\theta_1(|\epsilon|t|it)}&\times\\ \times[\theta_3^3(0|it)\theta_3(|\epsilon|t|it)+f(\Delta)\lambda\theta_4^3(0|it)\theta_4(|\epsilon|t|it)&\\ -\theta_2^3(0|it)\theta_2(|\epsilon|t|it)]\,,&\end{aligned} \tag{3.20}$$

where we restore the dependence on $2\pi\alpha'$. In the above relation, $q = \mathrm{e}^{-2\pi t}$, $\theta_i(\nu|\tau), i = 1,2,3,4$ are theta functions, and $\eta(\tau)$ is Dedekind eta function. The parameter λ characterize the GSO projection, being 1 for D3 or -1 for $\bar{\text{D}}$3. The φ_{flux} is a real algebraic function of fluxes,

$$\varphi_{\text{flux}} \equiv \frac{D}{64\pi^4[E_1(1-E_1^2-E_2^2+B_2^2+B_3^2)-E_2B_1B_2]}\,, \tag{3.21}$$

where D is defined in (C.3). The φ_{flux} comes from several sources. One part of it is from the integral of zero-modes, because $C_0^\mu = -2\alpha' p^\mu$, $\mu = 0,1$. And we need to multiply factor $\frac{i}{[x^0,x^1]}$ on V_2 in $\mathcal{A}$ for a noncommutative normalization. This normalization factor contributes to φ_{flux}. The other numerical factors to φ_{flux} come from orientation, integral measure and GSO projection.

When there exist complex modes, there are open string pairs production. The creation rate ω could be read from the 1-loop vacuum amplitude [16]

$$\omega = -2\text{Im}(\frac{i}{V_2}\mathcal{A}) = -\frac{2\mathcal{A}}{V_2} \tag{3.22}$$

Since there exist poles in the integrand of $\mathcal{A}$ at $t = \frac{l}{|\epsilon|}(l = 1,2,\ldots)$, the contour integration give us nonvanishing ω. When $\Delta > 0$,

$$\begin{aligned}\omega = -\frac{\varphi_{\text{flux}}}{\alpha'^2}\sum_{l=1}^\infty\frac{|\epsilon|}{l^2}\mathrm{e}^{-\frac{y^2 l}{2\pi\alpha'|\epsilon|}}\frac{1}{\eta^{12}(i\frac{l}{|\epsilon|})}&\\ \times\begin{cases}((-1)^l-1)\theta_2^4(0|i\frac{l}{|\epsilon|}) & \text{D1}-\text{D3}\,,\\ ((-1)^l-1)\theta_3^4(0|i\frac{l}{|\epsilon|})+((-1)^l+1)\theta_4^4(0|i\frac{l}{|\epsilon|}) & \text{D1}-\bar{\text{D}}3\,.\end{cases}&\end{aligned} \tag{3.23}$$

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When $\Delta < 0$, the conclusion is opposite.

When $|\epsilon|$ are very small, from the asymptotic behavior of θ and η functions, we have

$$\frac{((-1)^l-1)\theta_2^4\left(0|i\frac{l}{|\epsilon|}\right)}{\eta^{12}\left(i\frac{l}{|\epsilon|}\right)} \sim 16((-1)^l-1)\left(1+O\left(\mathrm{e}^{-\frac{2\pi l}{|\epsilon|}}\right)\right) \tag{3.24}$$

and

$$\frac{((-1)^l-1)\theta_3^4\left(0|i\frac{l}{|\epsilon|}\right)+((-1)^l+1)\theta_4^4\left(0|i\frac{l}{|\epsilon|}\right)}{\eta^{12}\left(i\frac{l}{|\epsilon|}\right)} \sim 2\mathrm{e}^{\frac{\pi l}{|\epsilon|}}(-1)^l\left(1+O\left(\mathrm{e}^{-\frac{\pi l}{|\epsilon|}}\right)\right). \tag{3.25}$$

Therefore, we learn that when two branes are far away from each other, the contribution from $l = 1$ dominate. When two branes move to each other, the contribution from higher values l become more and more important. Since $|\epsilon| \approx 0$, for D1-D3 case with $\Delta > 0$, the open string pair production is exponentially suppressed. This is consistent with the fact that the system is now near-BPS. On the other hand, for D1-$\bar{\text{D}}$3 with $\Delta > 0$, if y is finite, it may suppress the creation of open string pair production, while if $y \approx 0$, the pair creation is enhanced, especially for large l. This indicates that the system is far from being supersymmetric.

3.2 GSO projection and near massless states

In this subsection, we will determine the GSO projection in NS sector, and prove (3.19). We will also study the spectrum of open strings in the near-BPS case. This happens when the excitation modes are real and the fluxes are taken to be in a decoupling limit.

In last subsection, we find that the eigenvalue of GSO projection operator on $|0\rangle_{\mathrm{NS}}$ is a function $\frac{1+f(\Delta)}{2}$, in which $f(\Delta)$ can only be ± 1. The value of $f(\Delta)$ depends on the sign of Δ, and take opposite values when Δ change sign. To determine GSO projection, we analyze $\epsilon \to 0$ limit of one-loop amplitude $\mathcal{A}$. From the definition of ϵ, we know that $D \to 0$, so $\varphi_{\mathrm{flux}} \to 0$ at the zero ϵ limit. However, at the same time $\theta_1(|\epsilon|t|it) \to 0$ too. Using the definition of theta function, we get (for D1-D3)

$$\begin{aligned}\lim_{\epsilon\to 0}\mathcal{A} = {}& i\frac{|\Delta|V_2}{128\pi^4\alpha'^2[E_1(1-E_1^2-E_1^2+B_2^2+B_3^2)-E_2B_1B_2]}\times \\ &\times\int_0^\infty dt\frac{q^{\frac{y^2}{4\pi^2\alpha'}}}{t^3\eta^{12}(it)}\left[\theta_3^4(0|it)+f(\Delta)\theta_4^4(0|it)-\theta_2^4(0|it)\right]\ .\end{aligned} \tag{3.26}$$

In section 2 and appendix B, we have already obtained all supersymmetric conditions for D1-D3 systems with fluxes. These supersymmetric conditions are equivalent to

$$\begin{aligned}&D = 0\,,\\ &\Delta > 0\,.\end{aligned} \tag{3.27}$$

In the supersymmetric case, one-loop amplitude must be zero. So $\lim_{\epsilon\to 0}\mathcal{A}$ must be zero when $\Delta > 0, D = 0$. Recall that theta function satisfies Jacobi's 'abstruse identity'

$$\theta_3^4(0|it)-\theta_4^4(0|it)-\theta_2^4(0|it) = 0\,, \tag{3.28}$$

so $f(\Delta)$ must be -1 when $\Delta > 0$. This is just (3.19). Similarly, we can determine the GSO projection in D1-$\bar{\text{D}}$3, which is just changing $f(\Delta)$ to $-f(\Delta)$.

Let us discuss the case when $\tilde{E} = E_1$ and $D > 0$. From (3.8), we know that there are real fractional modes in this case. Let us introduce real parameters

$$\tilde{D} \equiv \sqrt{\Lambda}\,, \tag{3.29}$$

and

$$\tilde{\epsilon} \equiv \frac{1}{\pi}\arctan\frac{\tilde{D}}{\Delta}\ . \tag{3.30}$$

We can obtain the mode expansions and relations of modes like (C.1), (C.4), (C.2) and (C.5), by replacing $i\epsilon$, D with $\tilde{\epsilon}$, $-i\tilde{D}$ respectively.

The one-loop amplitude $\tilde{\mathcal{A}}$ is now

$$\begin{aligned}\tilde{\mathcal{A}} = \frac{\tilde{\varphi}_{\text{flux}}}{\alpha'^2}V_2\int_0^\infty dt\frac{q^{\frac{y^2}{4\pi^2\alpha'}}}{t^2\eta^9(it)\theta_1(-i|\tilde{\epsilon}|t|it)}\times \\ \times[\theta_3^3(0|it)\theta_3(-i|\tilde{\epsilon}|t|it)+f(\Delta)\lambda\theta_4^3(0|it)\theta_4(-i|\tilde{\epsilon}|t|it) \\ -\theta_2^3(0|it)\theta_2(-i|\tilde{\epsilon}|t|it)]\ ,\end{aligned} \tag{3.31}$$

where

$$\tilde{\varphi}_{\text{flux}} \equiv \frac{\tilde{D}}{64\pi^4[E_1(1-E_1^2-E_2^2+B_2^2+B_3^2)-E_2B_1B_2]}\ . \tag{3.32}$$

$f(\Delta)$ is defined as (3.19) too. The function in the integrand of (3.31) now are pure imaginary and only have pole at $t=0$ on positive real axis. Thus there is no open string pair production in this case.

In this case, it is meaningful to discuss the mass spectrum of open string between D1 and D3-brane. In D1-D3 systems, if we let $y=0$, the ground state in NS sector has energy

$$\mathcal{E}_v = \frac{|\tilde{\epsilon}|}{2} - \frac{1}{2}\ . \tag{3.33}$$

When $\Delta > 0$, the excited states $\beta_{-\frac{1}{2}\pm\tilde{\epsilon}}|0\rangle_{\text{NS}}$ have energies $-\frac{\tilde{\epsilon}}{2}$ and $\frac{3\tilde{\epsilon}}{2}$ respectively. Now under GSO projection, the ground state $|0\rangle_{\text{NS}}$ is projected out while $\beta_{-\frac{1}{2}\pm\tilde{\epsilon}}|0\rangle_{\text{NS}}$ survive. When $\tilde{\epsilon}$ is very small, these states become near massless. Other than this, the states $\beta^{\mu}_{-\frac{1}{2}}|0\rangle_{\text{NS}} \equiv \beta^{\mu+}_{-\frac{1}{2}}|0\rangle_{\text{NS}}$, $\mu = 4,5,\ldots,9$ all have energies $\frac{\tilde{\epsilon}}{2}$, they are all near massless when $\tilde{\epsilon}$ is very small. Furthermore, one can act on these states with an arbitrary polynomial consisting of $b_{-\tilde{\epsilon}}$ with energy $\tilde{\epsilon}$. This action gives rise to a large number of near massless states.

The configurations with $\tilde{\epsilon} \approx 0$ are called near-BPS. This happens when $\tilde{D} \approx 0$ or $\Delta \to \infty$. The solutions of $\tilde{D} = 0$ are (2.20) and (2.21), which are the supersymmetric conditions. It is expected that when $\tilde{D} \approx 0$ there are many near massless states. On the other hand, the fact that the case with $\Delta \to \infty$ has many near massless states sounds strange. One way to understand this fact is to take a large B_1 to get a large Δ. Effectively we can neglect other fluxes and simplify our system to D1-D3 with a large magnetic field

B_1. For this simplified system, it has been known to be near-BPS and has many near-massless states [11]. In this case, the magnetic field on $D3$ may induce a large number D1's so the system is near-BPS. In fact, one can understand this configuration from a dual description in matrix model [10].

However, still for D1-D3 systems, when $\Delta < 0$, the picture is very different. Because now the ground state $|0\rangle_{\rm NS}$ survive GSO projection and the first excited states are all projected out, the tachyon is there though we take $\tilde{\epsilon} \to 0$ limit. Now the system is far from supersymmetry.

For D1-$\bar{\rm D}$3 systems, the conclusions are opposite. When $\Delta < 0$, there are many near massless states when $\tilde{\epsilon} \to 0$. And when $\Delta > 0$, there is no state become near massless when $\tilde{\epsilon} \to 0$.

To get the near-BPS configuration, we take the large B limit and keep other fluxes to be finite. However, one has to be careful about the limit when $E_1 \to 1$. When $E_1 = 1$, this gives a noncommutative open string theory for D-string. Actually this extremal electric field washes out the other fluxes on D3-brane. And the whole D1-D3 system seems to be stable and supersymmetric. But when E_1 is not exactly extremal, it competes with the influence of large B_1 and the end result is that Δ is finite so that the configuration is not near-BPS.

4. Conclusions and discussions

In this paper, we studied D1-D3 (or $\bar{\rm D}$3) systems with constant fluxes in flat spacetime. We worked out all configurations which keep one-quarter supersymmetries. The result were summarized at the end of section 2. The supersymmetric configurations are T-dual to the D0-D2 brane system, and dual to supersymmetric intersecting D1-D1 with relative angle and motion, which has been studied in [5]. Furthermore, we investigated generic nonsupersymmetric configurations by quantizing the open string between D1 and D3 (or $\bar{\rm D}$3). In general, the open string modes could be complex or real fractional, rather than integer or half-integer. When the modes are complex, we obtained the open string pair production rate from 1-loop amplitude. When the modes are real, we discussed the open string mass spectrum and found that there could exist a large number of near massless states when the system is near-BPS. This is reminiscent of the same phenomenon discussed in [11, 12].

Our study of fluxed D1-D3 (or $\bar{\rm D}$3) system shows that turning on background fluxes can recover the supersymmetries of a non-BPS system. Generically speaking, it is quite difficult to decide if a system with fluxes is supersymmetric or not, since the supersymmetry analysis is quite involved. In particular, when the dimensionality of D-brane gets large, the number of possible background fluxes are large so that it is not easy to work out all the supersymmetric configurations. It would be interesting to find a more effective way to solve the problem.

From our study, it turned out that the GSO projection is quite subtle in the study of open string excitation, especially when there are background fluxes. We decide the GSO projection by taking the BPS limit. This would be a nice way to determine the GSO projection in more general setting.

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In the study of the open string spectrum, we noticed that there would be large number of light states if the system is near-BPS. One way to reach near-BPS configuration is to let the magnetic field in the codimendion be large. Effectively one can neglect other fluxes in the system and the system is reduced to the ones studied in [11]. This picture will be true for other systems.

In this article, we do not discuss the case that the fluxes take the critical values. It was found in [18] that when the electric field take the critical value, one can define a novel string theory. This string theory is an interacting open string theory, in which the close strings decouple from the open ones. In our case, if we let $\tilde{E} = E_1 = 1$, we are actually discussing the fluxed D3-branes in a noncommutative open string theory. Naively, from (3.8), it seems that only integer bosonic modes can exist, which indicates that the configurations is supersymmetric no matter what kind of fluxes we turn on the D3-brane. There would be no open string pair production or tachyon condensation, as the case discussed in other non-BPS system with critical electric field [19]. We look forward to a rigorous discussion about the critical fluxes in this system.

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A. Solutions of (2.19)

Here we will analyze the possible solutions of (2.19). Firstly, the second equation of (2.19) can be factorized into

$$(\tilde{E} - E_1)[(\tilde{E}E_1 - 1)B_1 + \tilde{E}E_2B_2] = 0. \tag{A.1}$$

One solution is $\tilde{E} = E_1$. If so, the first equation of (2.19) is

$$-(1 - E_1^2)^2 + (1 - E_1^2)(E_2^2 - B_2^2 - B_3^2) + E_2^2B_2^2 = 0. \tag{A.2}$$

1. If $1 - E_1^2 - E_2^2 \neq 0$, we obtain (2.20). ($B_1 \neq \frac{E_1E_2B_2}{1-E_1^2}$ come from (2.6))
2. If $1 - E_1^2 - E_2^2 = 0$, eq. (A.2) now is

$$(1 - E_1^2)B_3^2 = 0. \tag{A.3}$$

Since we require $1 - E_1^2 \neq 0$, so $B_3 = 0$. This is (2.21). ($B_1 \neq \frac{E_1B_2}{E_2}$ come from (2.6))

When $\tilde{E} \neq E_1$, from (A.1),

$$(\tilde{E}E_1 - 1)B_1 + \tilde{E}E_2B_2 = 0. \tag{A.4}$$

If $\tilde{E}E_1 - 1 = 0$, we must let $E_2 = 0$ or $B_2 = 0$. If $\tilde{E}E_1 - 1 = 0, E_2 = 0$, the first equation of (2.19) is

$$-\left(1 - \frac{1}{E_1^2}\right)(B_2^2 + B_3^2) + \left(E_1 - \frac{1}{E_1}\right)^2 B_1^2 = 0, \tag{A.5}$$

which require

$$B_2^2 + B_3^2 = (E_1^2 - 1)B_1^2. \tag{A.6}$$

Then the left hand side of (2.6) equal to $1 - E_1^2$. From (2.5) and $\tilde{E}E_1 - 1 = 0$, we know $1 - E_1^2 < 0$, so (2.6) cannot be satisfied. To let $B_2 = 0$ lead to the same conclusion. Therefore there is no solution when $\tilde{E} \neq E_1, \tilde{E}E_1 - 1 = 0$.

When $\tilde{E} \neq E_1, \tilde{E}E_1 - 1 \neq 0$, we deduce from (A.4) that

$$B_1 = \frac{\tilde{E}E_2B_2}{1 - \tilde{E}E_1}. \tag{A.7}$$

Because (2.5), (2.6), we obtain

$$\begin{aligned} 1 - \tilde{E}^2 &> 0, \\ E_2^2 - B_2^2 - B_3^2 &< 1 - E_1^2 + B_1^2 - (E_1B_1 + E_2B_2)^2 \end{aligned} \tag{A.8}$$

So the right hand side of the first equation of (2.19) satisfy

$$\begin{aligned} -(\tilde{E}E_1 - 1)^2 &+ (1 - \tilde{E}^2)(E_2^2 - B_2^2 - B_3^2) + (\tilde{E}B_1 - E_1B_1 - E_2B_2)^2 \\ &< -(\tilde{E}E_1 - 1)^2 + (1 - \tilde{E}^2)(1 - E_1^2 + B_1^2 - (E_1B_1 + E_2B_2)^2) \\ &\quad + (\tilde{E}B_1 - E_1B_1 - E_2B_2)^2 \\ &= -(\tilde{E}E_1 - 1)^2 + (1 - \tilde{E}^2)(1 - E_1^2) \\ &\quad + (1 - \tilde{E}^2)(B_1^2 - (E_1B_1 + E_2B_2)^2) + (\tilde{E}B_1 - (E_1B_1 + E_2B_2))^2 \\ &= -(\tilde{E} - E_1)^2 + (B_1 - \tilde{E}(E_1B_1 + E_2B_2))^2. \end{aligned} \tag{A.9}$$

From (A.7), we obtain

$$\tilde{E}(E_1B_1 + E_2B_2) = B_1. \tag{A.10}$$

So

$$-(\tilde{E} - E_1)^2 + (B_1 - \tilde{E}(E_1B_1 + E_2B_2))^2 = -(\tilde{E} - E_1)^2 < 0. \tag{A.11}$$

From (A.9) and (A.11), we learn that the first equation of (2.19) can not be satisfied. So we prove that (2.5), (2.6) are in contradiction with (2.19) when $\tilde{E} \neq E_1, \tilde{E}E_1 - 1 \neq 0$.

In summary, when $\tilde{E} \neq E_1$, (2.19) have no solutions which do not break (2.5) and (2.6).

Therefore, the solutions (2.20) and (2.21) are all possible solutions of (2.19) which obey (2.5) and (2.6).

B. T-dual discussions

T-duality is a powerful technique for the study of D-branes. The different D-brane systems could be related to each other by T-duality. Shortly speaking, it exchange Neumann and Dirichlet boundary conditions [1, 2] of open string. One nice property of T-duality is that it keeps supersymmetry.

For a string ending on D-branes with fluxes, the boundary conditions is [15]

$$G_{\mu\nu}\partial_\sigma X^\nu + iF_{\mu\nu}\partial_t X^\nu = 0. \tag{B.1}$$

For general D1 − D3 system with fluxes (2.1) and (2.2), the boundary conditions of string ending on D1 are

$$\begin{aligned} &\partial_\sigma X^0 + i\tilde{E}\partial_t X^1 = 0, \\ &\partial_\sigma X^1 + i\tilde{E}\partial_t X^0 = 0, \end{aligned} \tag{B.2}$$

and the boundary conditions of string ending on D3 are

$$\begin{aligned} 0 &= \partial_\sigma X^0 + iE_1\partial_t X^1 + iE_2\partial_t X^2, \\ 0 &= \partial_\sigma X^1 + iE_1\partial_t X^0 + \mathrm{i}B_3\partial_t X^2 - iB_2\partial_t X^3, \\ 0 &= \partial_\sigma X^2 + iE_2\partial_t X^0 - iB_3\partial_t X^1 + iB_1\partial_t X^3, \\ 0 &= \partial_\sigma X^3 + iB_2\partial_t X^1 - iB_1\partial_t X^2, \end{aligned} \tag{B.3}$$

Now let us do T-dual in X^1 direction. The boundary conditions (B.2) and (B.3) are changed to

$$\begin{aligned} 0 &= \partial_\sigma(X^0 - \tilde{E}X^1), \\ 0 &= \partial_t(X^1 - \tilde{E}X^0), \end{aligned} \tag{B.4}$$

and

$$\begin{aligned} 0 &= \partial_\sigma(X^0 - E_1X^1) + iE_2\partial_t X^2, \\ 0 &= \partial_t(X^1 - E_1X^0 - B_3X^2 + B_2X^3), \\ 0 &= \partial_\sigma(X^2 + B_3X^1) + iE_2\partial_t X^0 + iB_1\partial_t X^3, \\ 0 &= \partial_\sigma(X^3 - B_2X^1) - iB_1\partial_t X^2. \end{aligned} \tag{B.5}$$

Let us first consider the solution (2.20). Defining

$$X^{0'} = \frac{X^0 - E_1X^1}{\sqrt{1-E_1^2}}, \qquad X^{1'} = \frac{X^1 - E_1X^0}{\sqrt{1-E_1^2}}, \tag{B.6}$$

then we get the boundary conditions

$$0 = \partial_\sigma X^{0'}, \qquad 0 = \partial_t X^{1'}, \tag{B.7}$$

and

$$\begin{aligned} 0 &= \partial_\sigma X^{0'} + i\hat{E}_2\partial_t X^2, \\ 0 &= \partial_t(X^{1'} - \hat{B}_3X^2 + \hat{B}_2X^3), \\ 0 &= \partial_\sigma(X^2 + \hat{B}_3X^{1'}) + i\hat{E}_2\partial_t X^{0'} + i\hat{B}_1\partial_t X^3, \\ 0 &= \partial_\sigma(X^3 - \hat{B}_2X^{1'}) - i\hat{B}_1\partial_t X^2, \end{aligned} \tag{B.8}$$

where

$$\begin{aligned} &\hat{E}_2 \equiv \frac{E_2}{\sqrt{1-E_1^2}}, &&\hat{B}_1 \equiv B_1 - \frac{E_1E_2B_2}{1-E_1^2}, \\ &\hat{B}_2 \equiv \frac{B_2}{\sqrt{1-E_1^2}}, &&\hat{B}_3 \equiv \frac{B_3}{\sqrt{1-E_1^2}}. \end{aligned} \tag{B.9}$$

After doing T-duality on $X^{1'}$, we come back to D1-D3 system but now with fluxes

$$\hat{\hat{E}} = \hat{E}_1 = 0,\ 1 < \hat{E}_2^2 < 1 + \hat{B}_3^3,\ \hat{B}_1 \neq 0,\ \hat{B}_2^2 = \frac{1 - \hat{E}_2^2 + \hat{B}_3^2}{\hat{E}_2^2 - 1} \tag{B.10}$$

In section 2, we prove this system is supersymmetric if $\hat{B}_1 > 0$. Because T-duality and Lorentz transformation do not change the number of supersymmetries, we conclude that with fluxes satisfy (2.20) and $B_1 - \frac{E_1 E_2 B_2}{1-E_1^2} = \hat{B}_1 > 0$, the original D1-D3 system preserve $\frac{1}{4}$ supersymmetry.

Similar discussions can do for D1-$\bar{\text{D}}$3 systems. We find that with fluxes constrained by (2.20) and $B_1 - \frac{E_1 E_2 B_2}{1-E_1^2} < 0$, D1-$\bar{\text{D}}$3 system preserve $\frac{1}{4}$ supersymmetry.

Furthermore for D1-D3, if we do rotation

$$\begin{aligned} X^{1''} &= \frac{1}{\sqrt{1+\hat{B}_2^2+\hat{B}_3^2}}(X^{1'} - \hat{B}_3 X^2 + \hat{B}_2 X^3), \\ X^{2'} &= \frac{1}{\sqrt{1+\hat{B}_3^2}}(\hat{B}_3 X^{1'} + X^2), \\ X^{3'} &= \frac{\sqrt{\hat{E}_2^2 - 1}}{\hat{E}_2 \hat{B}_3 \sqrt{1+\hat{B}_3^2}}(-\hat{B}_2 X^{1'} + \hat{B}_2 \hat{B}_3 X^2 + (1+\hat{B}_3^2) X^3) \end{aligned} \tag{B.11}$$

(B.8) become

$$\begin{aligned} 0 &= \partial_\sigma X^{0'} + i\frac{\hat{E}_2}{\sqrt{1+\hat{B}_3^2}}\partial_t X^{2'} - i\frac{\hat{B}_2\sqrt{\hat{E}_2^2-1}}{\sqrt{1+\hat{B}_3^2}}\partial_t X^{3'}, \\ 0 &= \partial_t X^{1''}, \\ 0 &= \partial_\sigma X^{2'} + i\frac{\hat{E}_2}{\sqrt{1+\hat{B}_3^2}}\partial_t X^{0'} + i\frac{\hat{B}_1\sqrt{\hat{E}_2^2-1}}{\hat{E}_2\hat{B}_3}\partial_t X^{3'}, \\ 0 &= \partial_\sigma X^{3'} - i\frac{\hat{B}_2\sqrt{\hat{E}_2^2-1}}{\sqrt{1+\hat{B}_3^2}}\partial_t X^{0'} - i\frac{\hat{B}_1\sqrt{\hat{E}_2^2-1}}{\hat{E}_2\hat{B}_3}\partial_t X^{2'}. \end{aligned} \tag{B.12}$$

The system now becomes static D0-D2 system with constant fluxes. One can do another T-duality in $X^{2'}$ direction and change the system to intersecting D1-D1's with relative angle and motion.

In [5], the D2-D2 system with generic fluxes has been studied. The supersymmetric configurations found there could be dual to two intersecting D1's, which are moving relative to each other with angle. The supersymmetric condition is

$$-e_2^2(1-\beta_1^2)(1-\beta_2^2) + \sin^2\theta = \beta_1^2 + \beta_2^2 - 2\beta_1\beta_2\cos\theta. \tag{B.13}$$

In this equation, e_2 is the electric flux on the second D1, β_1, β_2 are normal speed of two D1s, θ is the angle between two strings.

In our case, after T-duality in $X^{2'}$ direction, the system now is a D1-D1 system with [7]

$$\beta_1 = 0, \qquad \beta_2 = \frac{\sin\theta \hat{E}_2}{\sqrt{1+\hat{B}_3^2}}, \qquad e_1 = 0, \qquad e_2 = -\frac{\sin\theta \hat{B}_2 \sqrt{\hat{E}_2^2 - 1}}{\sqrt{1-\beta_2^2}\sqrt{1+\hat{B}_3^2}},$$
$$\cot\theta = -\frac{\hat{B}_1\sqrt{\hat{E}_2^2-1}}{\hat{E}_2\hat{B}_3}, \tag{B.14}$$

where $e_1, e_2, \beta_1, \beta_2, \theta$ have the same meaning as the ones in (B.13). It is easy to check that the above identifications (B.14) satisfy the supersymmetric condition (B.13). This confirms that our supersymmetric analysis is correct.

For the solutions (2.21), let

$$X^{0'} = \frac{X^0 - E_1 X^1}{\sqrt{1-E_1^2}}, \qquad X^{1'} = \frac{X^1 - E_1 X^0}{\sqrt{1-E_1^2}}, \tag{B.15}$$

the boundary conditions (B.4) and (B.5) can be rewritten as

$$0 = \partial_\sigma X^{0'}, \qquad 0 = \partial_t X^{1'}, \tag{B.16}$$

and

$$\begin{aligned}
0 &= \partial_\sigma X^{0'} \pm i\partial_t X^2, \\
0 &= \partial_t\left(X^{1'} \pm \frac{B_2}{E_2}X^3\right), \\
0 &= \partial_\sigma X^2 \pm i\partial_t X^{0'} + i\left(B_1 - \frac{E_1B_2}{E_2}\right)\partial_t X^3, \\
0 &= \partial_\sigma\left(X^3 \mp \frac{B_2}{E_2}X^{1'}\right) - i\left(B_1 - \frac{E_1B_2}{E_2}\right)\partial_t X^2.
\end{aligned} \tag{B.17}$$

Similarly, we find that this system is T-dual to D1-D3 system with fluxes

$$\hat{\hat{E}} = \hat{E}_1 = 0, \qquad \hat{B}_1 = B_1 - \frac{E_1B_2}{E_2}, \qquad \hat{B}_2 = 0, \qquad \hat{B}_3 = 0,$$
$$\hat{E}_2 = \pm\sqrt{1-\hat{E}_1^2}. \tag{B.18}$$

We know this system preserve $\frac{1}{4}$ supersymmetry from section 2. Thus if the fluxes satisfy (2.21) and $B_1 - \frac{E_1B_2}{E_2} > 0$, the original D1-D3 system preserve $\frac{1}{4}$ supersymmetry. And with fluxes as (2.21) and $B_1 - \frac{E_1B_2}{E_2} < 0$, D1-$\bar{\text{D}}$3 system preserve $\frac{1}{4}$ supersymmetry.

For D1-D3 systems, if we do rotation

$$\begin{aligned}
X^{1''} &= \frac{X^{1'} \pm \frac{B_2}{E_2}X^3}{\sqrt{1+\frac{B_2^2}{E_2^2}}}, \\
X^{3'} &= \frac{X^3 \mp \frac{B_2}{E_2}X^{1'}}{\sqrt{1+\frac{B_2^2}{E_2^2}}}.
\end{aligned} \tag{B.19}$$

(B.17) equal to

$$\begin{aligned}
0 &= \partial_\sigma X^{0'} \pm i\partial_t X^2, \\
0 &= \partial_t X^{1''}, \\
0 &= \partial_\sigma X^2 \pm i\partial_t X^{0'} + i\frac{1}{\sqrt{1+\frac{B_2^2}{E_2^2}}}\left(B_1 - \frac{E_1 B_2}{E_2}\right)\partial_t X^{3'}, \\
0 &= \partial_\sigma X^{3'} - i\frac{1}{\sqrt{1+\frac{B_2^2}{E_2^2}}}\left(B_1 - \frac{E_1 B_2}{E_2}\right)\partial_t X^2.
\end{aligned} \tag{B.20}$$

This system becomes a D0-D2 system with fluxes.

We now do another T-duality in $X^{3'}$ direction for the D0-D2 system mentioned above. The system becomes intersecting D1-D1 system with

$$\beta_1 = \beta_2 = 0,\ e_1 = 0,\ e_2 = \sin\theta,\ \cot\theta = \frac{1}{\sqrt{1+\frac{B_2^2}{E_2^2}}}\left(B_1 + \frac{E_1 B_2}{E_2}\right). \tag{B.21}$$

It is easy to see that (B.21) satisfy the supersymmetric condition (B.13).

C. Mode expansion and quantization

When $\Lambda < 0$, the mode expansion for X^1, X^2, X^3 with all possible modes are

$$\begin{aligned}
X^\mu &= x_0^\mu + B_0^\mu \sigma - C_0^\mu \tau + \sum_{n\neq 0} \frac{i a_n^\mu}{n}(\mathrm{e}^{-in(\tau-\sigma)} + \mathrm{e}^{-in(\tau+\sigma)}) \\
&\quad + \sum_{n\neq 0} \frac{i b_n^\mu}{n}(\mathrm{e}^{-in(\tau-\sigma)} - \mathrm{e}^{-in(\tau+\sigma)}) \\
&\quad + \sum_{n+i\epsilon} \frac{i a_{n+i\epsilon}^\mu}{n+i\epsilon}(\mathrm{e}^{-i(n+i\epsilon)(\tau-\sigma)} + \mathrm{e}^{-i(n+i\epsilon)(\tau+\sigma)}) \\
&\quad + \sum_{n+i\epsilon} \frac{i b_{n+i\epsilon}^\mu}{n+i\epsilon}(\mathrm{e}^{-i(n+i\epsilon)(\tau-\sigma)} - \mathrm{e}^{-i(n+i\epsilon)(\tau+\sigma)}) \\
&\quad + \sum_{n-i\epsilon} \frac{i a_{n-i\epsilon}^\mu}{n-i\epsilon}(\mathrm{e}^{-i(n-i\epsilon)(\tau-\sigma)} + \mathrm{e}^{-i(n-i\epsilon)(\tau+\sigma)}) \\
&\quad + \sum_{n-i\epsilon} \frac{i b_{n-i\epsilon}^\mu}{n-i\epsilon}(\mathrm{e}^{-i(n-i\epsilon)(\tau-\sigma)} - \mathrm{e}^{-i(n-i\epsilon)(\tau+\sigma)})
\end{aligned} \tag{C.1}$$

Not all the coefficients of these modes are nonzero or independent. From the boundary conditions (3.4) and (3.5), we can find that there are following relations

$$\begin{aligned}
x_0^2 &= x_0^3 = C_0^2 = C_0^3 = a_n^2 = a_n^3 = a_{n\pm i\epsilon}^2 = a_{n\pm i\epsilon}^3 = 0. \\
B_0^0 &= E_1 C_0^1,\ B_0^1 = E_1 C_0^0,\ B_0^2 = E_2 C_0^0 - B_3 C_0^1,\ B_0^3 = B_2 C_0^1.
\end{aligned}$$

$$
\begin{aligned}
b_n^0 &= E_1 a_n^1,\ b_n^1 = E_1 a_n^0,\ b_n^2 = E_2 a_n^0 - B_3 a_n^1,\ b_n^3 = B_2 a_n^1. \\
b_{n\pm i\epsilon}^0 &= E_1 a_{n\pm i\epsilon}^1,\ b_{n\pm i\epsilon}^1 = E_1 a_{n\pm i\epsilon}^0 \\
a_{n\pm i\epsilon}^0 &= \frac{E_2}{1-E_1^2} b_{n\pm i\epsilon}^2,\ a_{n\pm i\epsilon}^1 = \frac{(1-E_1^2)B_3 \mp B_2 D}{(1-E_1^2)(1-E_1^2+B_2^2)} b_{n\pm i\epsilon}^2, \\
b_{n\pm i\epsilon}^3 &= \frac{B_2 B_3 \pm D}{1-E_1^2+B_2^2} b_{n\pm i\epsilon}^2.
\end{aligned} \tag{C.2}
$$

Here D is defined as

$$
D \equiv \sqrt{-\Lambda}\ . \tag{C.3}
$$

And $x_0^0, x_0^1, C_0^0, C_0^1, a_n^0, a_n^1, b_{n\pm i\epsilon}^2$ are independent. We know D is real since $\Lambda < 0$.

The mode expansion for the fermions can be obtained similary. The possible mode expansions are

$$
\begin{aligned}
\psi_+^\mu &= \sum_r \alpha_r^{\mu +} \mathrm{e}^{-ir(\tau+\sigma)} \\
&\quad + \sum_{r+i\epsilon} \beta_{r+i\epsilon}^{\mu+} \mathrm{e}^{-i(r+i\epsilon)(\tau+\sigma)} + \sum_{r-i\epsilon} \beta_{r-i\epsilon}^{\mu+} \mathrm{e}^{-i(r-i\epsilon)(\tau+\sigma)}, \\
\psi_-^\mu &= \sum_r \alpha_r^{\mu -} \mathrm{e}^{-ir(\tau-\sigma)} \\
&\quad + \sum_{r+i\epsilon} \beta_{r+i\epsilon}^{\mu-} \mathrm{e}^{-i(r+i\epsilon)(\tau-\sigma)} + \sum_{r-i\epsilon} \beta_{r-i\epsilon}^{\mu-} \mathrm{e}^{-i(r-i\epsilon)(\tau-\sigma)}.
\end{aligned} \tag{C.4}
$$

The coefficients of these modes have the following relations

$$
\begin{aligned}
\alpha_r^{0-} &= \frac{(1+E_1^2)\alpha_r^{0+} + 2E_1\alpha_r^{1+}}{1-E_1^2}, \alpha_r^{1-} = \frac{(1+E_1^2)\alpha_r^{1+} + 2E_1\alpha_r^{0+}}{1-E_1^2}, \\
\alpha_r^{2+} &= -\alpha_r^{2-} = -\frac{(E_2 - E_1 B_3)\alpha_r^{0+} + (E_1 E_2 - B_3)\alpha_r^{1+}}{1-E_1^2} \\
\alpha_r^{3+} &= -\alpha_r^{3-} = -\frac{E_1 B_2 \alpha_r^{0+} + B_2 \alpha_r^{1+}}{1-E_1^2}, \\
\beta_{r\pm i\epsilon}^{0+} &= \left(\frac{E_1(1-E_1^2)B_3 \mp E_1 B_2 D}{(1-E_1^2)(1-E_1^2+B_2^2)} - \frac{E_2}{1-E_1^2}\right)\beta_{r\pm i\epsilon}^{2+}, \\
\beta_{r\pm i\epsilon}^{0-} &= \left(-\frac{E_1(1-E_1^2)B_3 \mp E_1 B_2 D}{(1-E_1^2)(1-E_1^2+B_2^2)} - \frac{E_2}{1-E_1^2}\right)\beta_{r\pm i\epsilon}^{2+}, \\
\beta_{r\pm i\epsilon}^{1+} &= \left(\frac{E_1 E_2}{1-E_1^2} - \frac{(1-E_1^2)B_3 \mp B_2 D}{(1-E_1^2)(1-E_1^2+B_2^2)}\right)\beta_{r\pm i\epsilon}^{2+}, \\
\beta_{r\pm i\epsilon}^{1-} &= \left(-\frac{E_1 E_2}{1-E_1^2} - \frac{(1-E_1^2)B_3 \mp B_2 D}{(1-E_1^2)(1-E_1^2+B_2^2)}\right)\beta_{r\pm i\epsilon}^{2+}, \\
\beta_{r\pm i\epsilon}^{2-} &= -\beta_{r\pm i\epsilon}^{2+}, \\
\beta_{r\pm i\epsilon}^{3+} &= -\beta_{r\pm i\epsilon}^{3-} = \frac{B_2 B_3 \pm D}{1-E_1^2+B_2^2}\beta_{r\pm i\epsilon}^{2+}.
\end{aligned} \tag{C.5}
$$

Here r is integer (for R sector) or half integer (for NS sector). Definition of D can be found in (C.3). $\alpha_r^{0+}, \alpha_r^{1+}, \beta_{r\pm i\epsilon}^{2+}$ are independent.

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The symplectic form is defined as

$$\Omega = \int_0^\pi d\sigma(\delta\Pi_{X_\mu} \wedge \delta X^\mu - \delta\Pi_{\psi_\mu} \wedge \delta\psi^\mu), \tag{C.6}$$

where $\psi^\mu = (\psi^{\mu+}, \psi^{\mu-})$ is world-sheet Majorana spinor, and $\Pi_{X_\mu}, \Pi_{\psi_\mu}$ are the conjugate momenta of X^μ, ψ^μ

$$\Pi_{X_\mu} = \eta_{\mu\nu}\partial_\tau X^\mu + (A_\mu^{(0)}\delta(\sigma) - A_\mu^{(\pi)}\delta(\sigma-\pi)), \quad \Pi_{\psi_\mu} = \frac{i}{2}\bar{\psi}^\nu\gamma^0\eta_{\mu\nu}. \tag{C.7}$$

Here $\gamma^0 = i\sigma^2$ is a two-dimensional gamma matrix. $A_\mu^{(0)}, A_\mu^{(\pi)}$ are gauge potentials on D1,D3 brane, whose field strengths are (2.1), (2.2) respectively.

With the mode expansions (C.1) and (C.4), we can calculate Ω using (C.6). Because relations (C.2) and (C.5), we should write Ω in independent variables at final result. After integration and some algebraic calculations, we obtain

$$\begin{aligned}
\Omega = {} & \pi(E_1^2+E_2^2-1)\delta x_0^0 \wedge \delta C_0^0 + \pi(1-E_1^2+B_2^2+B_3^2)\delta x_0^1 \wedge \delta C_0^1 \\
& -\pi E_2 B_3(\delta x_0^0 \wedge \delta C_0^1 + \delta x_0^1 \wedge \delta C_0^0) \\
& +\pi^2(E_1(1-E_1^2-E_2^2+B_2^2+B_3^2) - E_2B_1B_2)\delta C_0^0 \wedge \delta C_0^1 \\
& +\sum_{n\neq 0}\frac{2\pi i}{n}(1-E_1^2-E_2^2)\delta a_n^0 \wedge \delta a_{-n}^0 \\
& -\sum_{n\neq 0}\frac{2\pi i}{n}(1-E_1^2+B_2^2+B_3^2)\delta a_n^1 \wedge \delta a_{-n}^1 \\
& +\sum_{n\neq 0}\frac{4\pi i}{n}E_2B_3\delta a_n^0 \wedge \delta a_{-n}^1 \\
& -\sum_n \frac{8\pi i}{n+i\epsilon}\left(1-\frac{E_2^2}{1-E_1^2}+\frac{B_3^2}{1-E_1^2+B_2^2}\right)\delta b_{n+i\epsilon}^2 \wedge \delta b_{-n-i\epsilon}^2 \\
& +\sum_r \pi i\left(-1+\frac{(E_2-E_1B_3)^2+E_1^2B_2^2}{(1-E_1^2)^2}\right)\delta\alpha_r^{0+} \wedge \delta\alpha_{-r}^{0+} \\
& +\sum_r \pi i\left(1+\frac{(E_1E_2-B_3)^2+B_2^2}{(1-E_1^2)^2}\right)\delta\alpha_r^{1+} \wedge \delta\alpha_{-r}^{1+} \\
& +\sum_r 2\pi i\frac{(E_2-E_1B_3)(E_1E_2-B_3)+E_1B_2^2}{(1-E_1^2)^2}\delta\alpha_r^{0+} \wedge \delta\alpha_{-r}^{1+} \\
& +\sum_r 4\pi i\left(1-\frac{E_2^2}{1-E_1^2}+\frac{B_3^2}{1-E_1^2+B_2^2}\right)\delta\beta_{r+i\epsilon}^{2+} \wedge \delta\beta_{-r-i\epsilon}^{2+}
\end{aligned} \tag{C.8}$$

From symplectic form Ω, we can obtain Poisson bracket (here is Dirac bracket) through usual way. Then we can work out result of quantization from Dirac bracket directly. They are

$$
\begin{aligned}
[x_0^0, x_0^1] &= i\frac{E_1(1-E_1^2-E_2^2+B_2^2+B_3^2)-E_2B_1B_2}{D^2},\\
[x_0^0, C_0^0] &= -i\frac{1-E_1^2+B_2^2+B_3^2}{\pi D^2},\\
[x_0^1, C_0^1] &= -i\frac{E_1^2+E_2^2-1}{\pi D^2},\\
[x_0^0, C_0^1] &= [x_0^1, C_0^0] = -i\frac{E_2B_3}{\pi D^2},\\
[C_0^0, C_0^1] &= 0,\\
[a_n^0, a_m^0] &= \frac{n}{4\pi}\frac{1-E_1^2+B_2^2+B_3^2}{D^2}\delta_{n,-m},\\
[a_n^1, a_m^1] &= \frac{n}{4\pi}\frac{E_1^2+E_2^2-1}{D^2}\delta_{n,-m},\\
[a_n^0, a_m^1] &= \frac{n}{4\pi}\frac{E_2B_3}{D^2}\delta_{n,-m},\\
[b_{n+i\epsilon}^2, b_{m-i\epsilon}^2] &= -\frac{n+i\epsilon}{8\pi}\frac{(1-E_1^2)(1-E_1^2+B_2^2)}{D^2}\delta_{n,-m},\\
\{\alpha_r^{0+}, \alpha_s^{0+}\} &= \frac{1}{2\pi}\frac{(1-E_1^2)^2+(E_1E_2-B_3)^2+B_2^2}{D^2}\delta_{r,-s},\\
\{\alpha_r^{1+}, \alpha_s^{1+}\} &= \frac{1}{2\pi}\frac{-(1-E_1^2)^2+(E_2-E_1B_3)^2+E_1^2B_2^2}{D^2}\delta_{r,-s},\\
\{\alpha_r^{0+}, \alpha_s^{1+}\} &= -\frac{1}{2\pi}\frac{(E_2-E_1B_3)(E_1E_2-B_3)+E_1B_2^2}{D^2}\delta_{r,-s},\\
\{\beta_{r+i\epsilon}^{2+}, \beta_{s-i\epsilon}^{2+}\} &= -\frac{1}{4\pi}\frac{(1-E_1^2)(1-E_1^2+B_2^2)}{D^2}\delta_{r,-s}.
\end{aligned}
\tag{C.9}
$$

As expectations, x_0^0 and x_0^1 are noncommutative. A little trouble is that $[a_n^0, a_{-n}^1]$, $\{\alpha_r^{0+}, \alpha_{-r}^{1+}\}$ are nonzero. 'Non-diagonal' (anti-)commutator will obstruct us to define Fock space. To resolve this problem, We transfer some modes. Linear transforms we will do for these modes can make 'Non-diagonal' commutator (anti-commutator) vanish. More over, as we have seen in section 3, those linear transforms can transfer world-sheet Hamiltonian to an elegant form. Details of our linear transforms are

$$
\begin{aligned}
c_n &\equiv -\frac{\sqrt{\frac{4\pi D^2}{1-E_1^2+B_2^2+B_3^2}}a_n^0+\sqrt{\frac{4\pi D^2}{E_1^2+E_2^2-1}}a_n^1}{\sqrt{2\left(1+\frac{E_2B_3}{\sqrt{(E_1^2+E_2^2-1)(1-E_1^2+B_2^2+B_3^2)}}\right)}},\\
d_n &\equiv -\frac{\sqrt{\frac{4\pi D^2}{1-E_1^2+B_2^2+B_3^2}}a_n^0-\sqrt{\frac{4\pi D^2}{E_1^2+E_2^2-1}}a_n^1}{\sqrt{2\left(1-\frac{E_2B_3}{\sqrt{(E_1^2+E_2^2-1)(1-E_1^2+B_2^2+B_3^2)}}\right)}},\\
b_{n\pm i\epsilon} &\equiv \sqrt{\frac{8\pi D^2}{(1-E_1^2)(1-E_1^2+B_2^2)}}b_{n\pm i\epsilon}^2,
\end{aligned}
$$

$$
\begin{aligned}
\phi_r &\equiv \frac{\sqrt{\frac{2\pi D^2}{(1-E_1^2)^2+(E_1E_2-B_3)^2+B_2^2}}\,\alpha_r^{0+}+\sqrt{\frac{2\pi D^2}{-(1-E_1^2)^2+(E_2-E_1B_3)^2+E_1^2B_2^2}}\,\alpha_r^{1+}}{\sqrt{2\left(1-\frac{(E_2-E_1B_3)(E_1E_2-B_3)+E_1B_2^2}{\sqrt{((1-E_1^2)^2+(E_1E_2-B_3)^2+B_2^2)(-(1-E_1^2)^2+(E_2-E_1B_3)^2+E_1^2B_2^2)}}\right)}},\\
\xi_r &\equiv \frac{\sqrt{\frac{2\pi D^2}{(1-E_1^2)^2+(E_1E_2-B_3)^2+B_2^2}}\,\alpha_r^{0+}-\sqrt{\frac{2\pi D^2}{-(1-E_1^2)^2+(E_2-E_1B_3)^2+E_1^2B_2^2}}\,\alpha_r^{1+}}{\sqrt{2\left(1+\frac{(E_2-E_1B_3)(E_1E_2-B_3)+E_1B_2^2}{\sqrt{((1-E_1^2)^2+(E_1E_2-B_3)^2+B_2^2)(-(1-E_1^2)^2+(E_2-E_1B_3)^2+E_1^2B_2^2)}}\right)}},\\
\beta_{r\pm i\epsilon} &\equiv \sqrt{\frac{4\pi D^2}{(1-E_1^2)(1-E_1^2+B_2^2)}}\,\beta_{r\pm i\epsilon}^{2+}. \qquad \text{(C.10)}
\end{aligned}
$$

After transforms (C.10), new operator satisfy commute(anti-commute) relations

$$
\begin{aligned}
[c_n, c_m] &= n\delta_{n,-m},\ [d_n, d_m] = n\delta_{n,-m},\ [c_n, d_m] = 0,\\
[b_{n+i\epsilon}, b_{m-i\epsilon}] &= -(n+i\epsilon)\delta_{n,-m}.\\
\{\phi_r, \phi_s\} &= \delta_{r,-s},\ \{\xi_r, \xi_s\} = \delta_{r,-s},\ \{\phi_r, \xi_s\} = 0,\\
\{\beta_{r+i\epsilon}, \beta_{s-i\epsilon}\} &= -\delta_{r,-s}. \qquad \text{(C.11)}
\end{aligned}
$$

C_0^μ should transfer like a_n^μ. But in this paper, the issue we concern undergo little influence from transforms of C_0^μ. So we do not write out their transforms explicitly.

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